Simultaneous Control of Flow and Pressure in a Coupled System

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1 Introduction

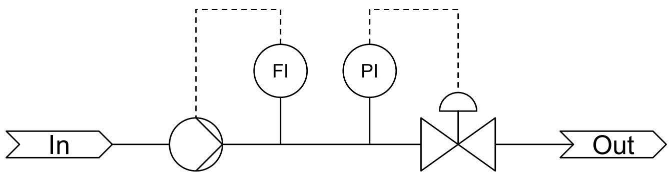

Figure 1: P&ID of flow-pressure control process.

Consider the system depicted in figure 1 comprising a pump with variable speed drive (VSD) upstream of a flow indicator, a pressure indication and a linear-characteristic control valve. The system is equipped with PI-controllers to simultaneously control the flow throughput and pressure. A model is developed which allows for simulation and model-based tuning of the two controllers.

A typical realization of this system is found in “high-temperature short-time” sterilization units in the bioprocessing industries. In this application ths VSD manipulates throughput while the control valve maintains the pressure at a value keeping the fluid in liquid form.

2 Model

The dynamics are separated into two: Those originating from the hydraulic dynamics in the fluid and those stemming from the mechanical dynamics in the pump and valve due to inertia. It is assumed that the mechanical dynamics are at a time scale much slower than the hydraulic and hence the hydraulic conditions are assumed to be in a pseudo steady state. Following this line of thought the system is conceptualized, as illustrated in figure 2, by two transfer functions which represent the dynamics of the mechanical components namely the pump and valve which interact with each other through the steady state mechanical energy balance of the fluid and the feed back controllers.

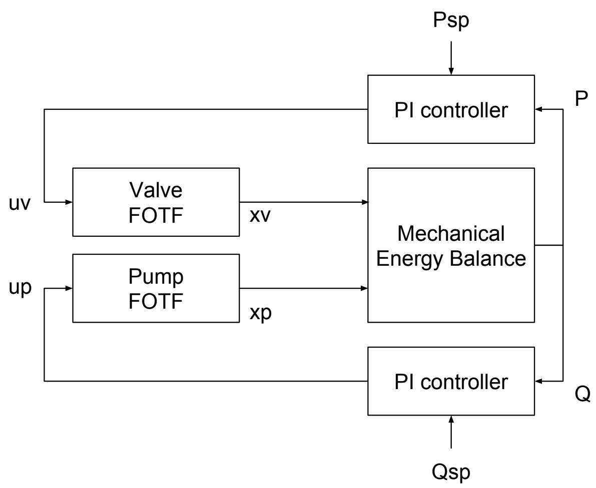

Figure 2: Conceptualization of the pump-valve system. First order transfer function is abbreviated as FOTF.

2.1 Pump and Valve Characteristics

The pump characteristic is approximated by a quadratic equation (1) where x_p is the pump rotational speed scaled between 0 and 100 percent.

$$P = x_p^2 \, P_{\max} - \alpha \, Q^2 \tag{1}$$

The valve characteristic is approximated by equation (2) where x_v is the degree to which the valve is open. It is assumed that the liquid is water.

$$x_v\,K_v = Q\sqrt{\frac{\mathrm{SG}}{\Delta p}} \tag{2}$$

The pressure drop across the valve is isolated yielding equation (3).

$$\Delta P = \left(\frac{Q}{x_v\,K_v}\right)^2 \tag{3}$$

Nominal equipment parameters are stated in table 1.

Table 1: Nominal system parameters

Parameter	Value
K_v	$4\,\mathrm{m^3\,h^{-1}\,bar^{-1/2}}$
SG	1
$P_{\max}$	$10\,\mathrm{bar}$
α	$0.1\,\mathrm{bar\,(m^3\,h^{-1})^{-2}}$

The characteristics, equations (1) and (3), are illustrated in figure 3 for varying pump input and valve input subject to the nominal parameters in table 1.

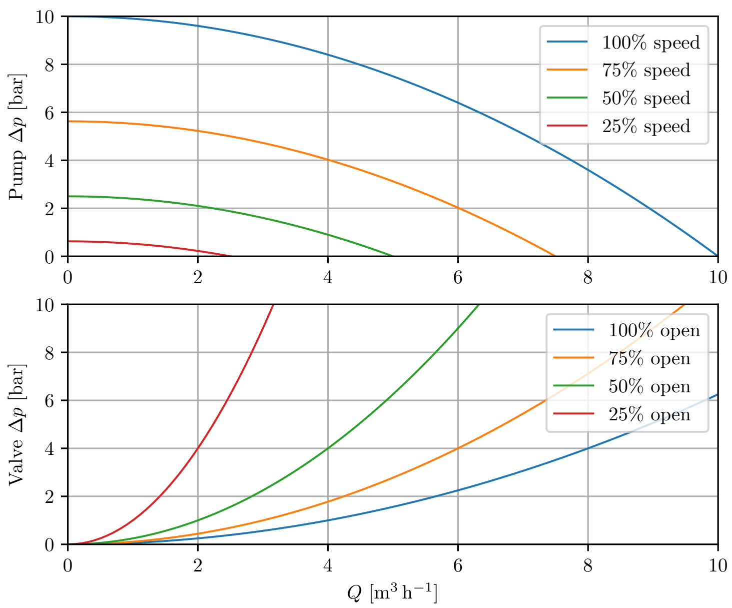

Figure 3: Pump and valve characteristics for varying inputs.

2.2 Mechanical Energy Balance

The steady state mechanical energy balance is given in equation (4).

$$\frac{\Delta P}{\rho} + \frac{v_2^2 - v_1^2}{2} = W - W_f \tag{4}$$

Inserting the pumpe and valve characteristics equations (1) and (3) and assuming negligible acceleration of the liquid yields equation (5).

$$P_{\text{out}} - P_{\text{in}} = x_p^2\, P_{\max} - \alpha\, Q^2 - \left(\frac{Q}{x_v\, K_v}\right)^2 \tag{5}$$

The output flow rate is then the solution to equation (6) for a reference pressure of $P_{\text{in}} = 0$:.

$$0 = x_p^2\, P_{\max} - P_{\text{out}} - \left(\alpha + \left[\frac{1}{x_v\, K_v}\right]^2\right) Q^2 \tag{6}$$

This solution is given in equation (7).

$$Q = \frac{\pm\sqrt{4\left(\alpha + \left[\frac{1}{x_v\, K_v}\right]^2\right)(x_p^2\, P_{\max} - P_{\text{out}})}}{2\left(\alpha + \left[\frac{1}{x_v\, K_v}\right]^2\right)} = \pm\sqrt{\frac{x_p^2\, P_{\max} - P_{\text{out}}}{\alpha + \left(\frac{1}{x_v\, K_v}\right)^2}} \tag{7}$$

It is noted that only the positive solution has a physical interpretation, hence:

$$Q = \sqrt{\frac{x_p^2\, P_{\max} - P_{\text{out}}}{\alpha + \left(\frac{1}{x_v\, K_v}\right)^2}} \tag{8}$$

The (Q, P)-envelope is presented in figure 4 which was calculated using the parameters in table 1. The hydraulic steady states of the system are confined to the interior of this envelope while points outside it exceed the specifications of the pump and valve. The upper bound is limited for conditions where the pump is fully on and the valve is partially open. The lower bound is limited for conditions where the valve is fully open and the pump is driven partially.

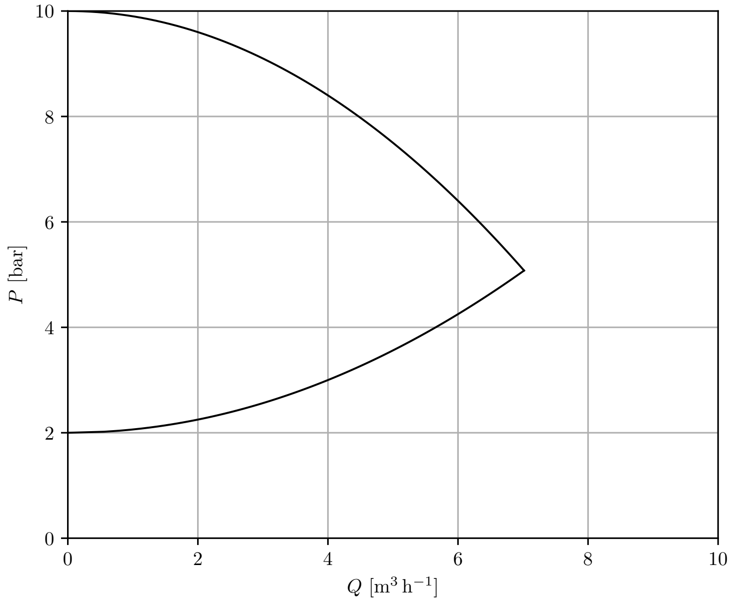

Figure 4: $(Q,\, P)$-envelope.

2.3 Dynamic Model

The system dynamics are captured by two first order transfer functions which give appropriate lags to the manipulation of the pump and valve simulating the mechanical inertia. The pressure and flow rate are calculated by the mechanical energy balance. The open-loop model is stated in the system of equations (9).

$$Q(t) = \sqrt{\frac{x_p(t)^2\, P_{\max} - P_{\text{out}}}{\alpha + \left(\frac{1}{x_v(t)\, K_v}\right)^2}} \tag{9a}$$

$$P(t) = x_p(t)^2\, P_{\max} - \alpha\, Q(t)^2 \tag{9b}$$

$$\frac{\mathrm{d}x_p}{\mathrm{d}t} = \frac{u_p - x_p(t)}{\tau_p} \tag{9c}$$

$$\frac{\mathrm{d}x_v}{\mathrm{d}t} = \frac{u_v - x_v(t)}{\tau_v} \tag{9d}$$

$$x_p(t=0) = x_{p,\,\mathrm{s}},\;\; x_v(t=0) = x_{v,\,\mathrm{s}} \tag{9e}$$

3 Control

In the following the pairing of manipulated variable and controlled variable is discussed. Following this the model is augmented with PI controllers and simulation results are presented.

3.1 MV/PV Pairing

The system of equations (9) may be linearized to give the system of equations (10).

$$
\begin{aligned}
Q(t) &= x_p(t)\, K_1 + x_v(t)\, K_2 && \text{(10a)}\\
P(t) &= x_p(t)\, K_3 + x_v(t)\, K_4 && \text{(10b)}\\
\frac{\mathrm{d}x_p}{\mathrm{d}t} &= \frac{u_p - x_p(t)}{\tau_p} && \text{(10c)}\\
\frac{\mathrm{d}x_v}{\mathrm{d}t} &= \frac{u_v - x_v(t)}{\tau_v} && \text{(10d)}\\
x_p(t=0) &= x_{p,\mathrm{s}},\ x_v(t=0) = x_{v,\mathrm{s}} && \text{(10e)}
\end{aligned}
$$

In this system K_1, K_2, K_3 and K_4 are the steady state gains as calculated by equation (11).

$$K_1 = \frac{\partial Q}{\partial x_p}\,,\ K_2 = \frac{\partial Q}{\partial x_v}\,,\ K_3 = \frac{\partial P}{\partial x_p}\,,\ K_4 = \frac{\partial P}{\partial x_v} \tag{11}$$

The system in (10) is a 2×2 system for which the Relative Gain Array (RGA) may be calculated by equation (12).

$$\mathrm{RGA} = \begin{bmatrix} \lambda & 1-\lambda \\ 1-\lambda & \lambda \end{bmatrix},\quad \lambda = \left(1 - \frac{K_2\, K_3}{K_1\, K_4}\right)^{-1} \tag{12}$$

The optimal control pairing is that which brings the eigenvalue of the RGA matrix, in this case λ close to a value of 1. It can be shown that λ is given by equation (13) which implies that the optimal PV-MV pairing depends on the pump, valve and vale operating point.

$$\lambda = \frac{1}{1 + \alpha\,(K_v\, x_{v,\mathrm{s}})^2} \tag{13}$$

The dependency of λ on $x_{v,\mathrm{s}}$ is illustrated in figure 5 using the parameters in table 1. The figure shows that one pairing is not ideal for all operating points. For $\lambda > 0.5$ the recommended pairing is valve-pressure/pump-flow while for $\lambda < 0.5$ the recommended pairing is the opposite. It should also be noted that equation (13) implies that all entries in the RGA matrix are positive.

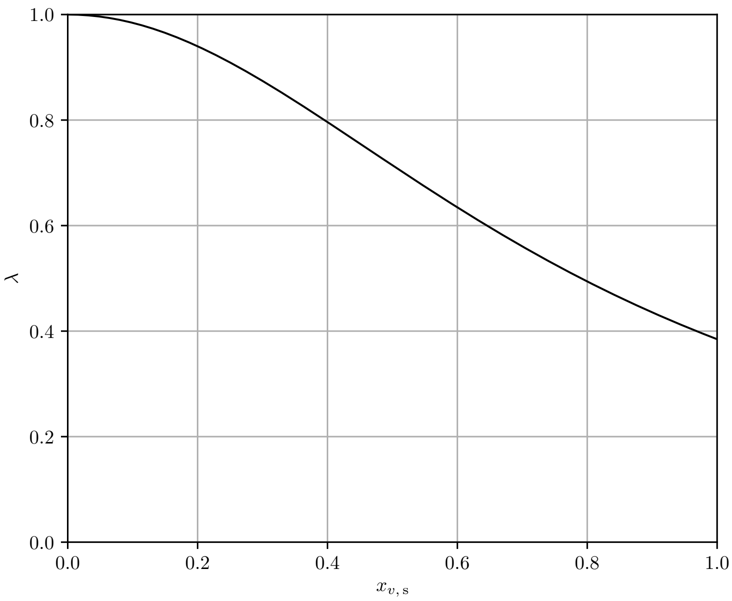

Figure 5: λ as a function of $x_{v,\,\mathrm{s}}$.

3.2 Closed loop model

The open loop model equation (9) is augmented with two PI controllers in accordance with figure 1 yielding the system of equations (14) for the closed loop case where Q_{sp} and P_{sp} are controller set points, I_p and I_v are controller integral states, $K_{\mathrm{p},\,p}$ and $K_{\mathrm{p},\,v}$ are proportional gains, $K_{\mathrm{I},\,p}$ and $K_{\mathrm{I},\,v}$ are integral gains, τ_p is the time constant of the pump dynamics and

τ_v is the time constant of the valve dynamics.

$$Q(t) = \sqrt{\frac{x_p(t)^2\,P_{\mathrm{max}} - P_{\mathrm{out}}}{\alpha + \left(\frac{1}{x_v(t)\,K_v}\right)^2}} \tag{14a}$$

$$P(t) = x_p(t)^2\,P_{\mathrm{max}} - \alpha\,Q(t)^2 \tag{14b}$$

$$\frac{\mathrm{d}I_p}{\mathrm{d}t} = Q_{\mathrm{sp}} - Q(t) \tag{14c}$$

$$u_p = x_{p,\,\mathrm{s}} + K_{\mathrm{p},\,p}((Q_{\mathrm{sp}} - Q(t)) + K_{\mathrm{I},\,p}\,I_p(t)) \tag{14d}$$

$$\frac{\mathrm{d}x_p}{\mathrm{d}t} = \frac{u_p - x_p}{\tau_p} \tag{14e}$$

$$\frac{\mathrm{d}I_v}{\mathrm{d}t} = P_{\mathrm{sp}} - P(t) \tag{14f}$$

$$u_v = x_{v,\,\mathrm{s}} + K_{\mathrm{p},\,v}((P_{\mathrm{sp}} - P(t)) + K_{\mathrm{I},\,v}\,I_v(t)) \tag{14g}$$

$$\frac{\mathrm{d}x_v}{\mathrm{d}t} = \frac{u_v - x_v}{\tau_v} \tag{14h}$$

$$x_p(t=0) = x_{p,\,\mathrm{s}},\; I_p(t=0) = 0,\; x_v(t=0) = x_{v,\,\mathrm{s}},\; I_v(t=0) = 0 \tag{14i}$$

3.3 Simulation

A dynamic simulation of equation (14) is shown in figure 6 where the system is subjected to a series of set point changes and a disturbance in the downstream pressure. The simulation indicates that satisfactory control can be obtained with PI controllers for the coupled system.

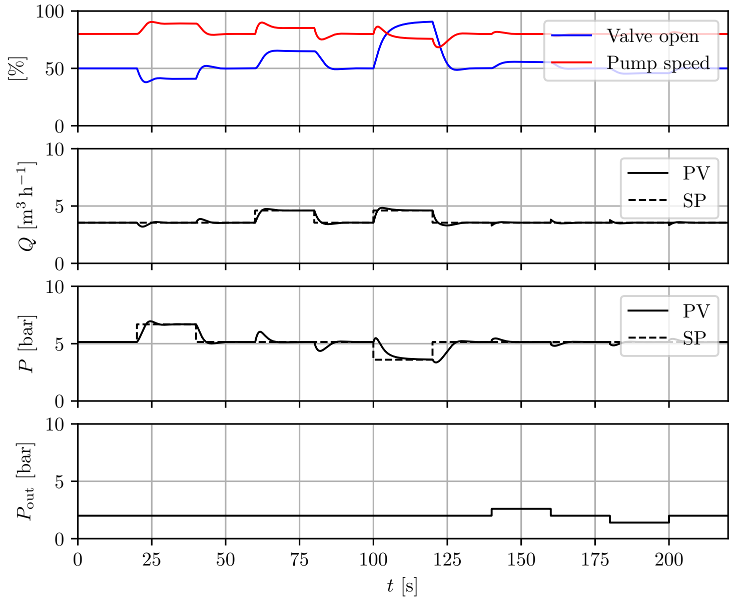

Figure 6: Simulation of equation (14) subject to set point changes and disturbances.